Trigonometry
Lecture 32

Unit Circle
Sin, Cos, Tan

Feb 19-8:47 AM

$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

(0,1)

(x,y)

(-1,0) θ (1,0)

(0,-1)

$\tan 0° = 0$ $\tan 30° = \frac{\sqrt{3}}{3}$ $\tan 45° = 1$ $\tan 60° = \sqrt{3}$ $\tan 90°$ und.

$-\frac{\pi}{2}$ $\frac{\pi}{2}$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$y = \tan x$ $(-\infty, \infty)$

Oct 21-10:29 AM

Graph $\quad y = \tan\left(x - \dfrac{\pi}{2}\right)$

$$-\dfrac{\pi}{2} < x - \dfrac{\pi}{2} < \dfrac{\pi}{2}$$

Add $\dfrac{\pi}{2}$ $\qquad$ $0 < x < \pi$

Oct 23-10:40 AM

Graph $\quad y = \tan\left(\dfrac{1}{2}x\right)$

$$-\dfrac{\pi}{2} < \dfrac{1}{2}x < \dfrac{\pi}{2}$$

multiply by 2

$$-\pi < x < \pi$$

Vertical Asymptote

Oct 23-10:42 AM

2

$$y = 1 + \tan\left(2x + \frac{\pi}{2}\right)$$

shift
it
up

1 unit.

$$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$$

$$-\pi < 4x + \pi < \pi$$

$$-2\pi < 4x < 0$$

$$\frac{-2\pi}{4} < x < \frac{0}{4}$$

$$-\frac{\pi}{2} < x < 0$$

$-\frac{\pi}{2}$ $-\frac{\pi}{4}$

Oct 23-10:45 AM

Graph $y = -\tan\left(\frac{1}{4}x - \frac{\pi}{4}\right)$

Reflection
about
x-axis

$$-\frac{\pi}{2} < \frac{1}{4}x - \frac{\pi}{4} < \frac{\pi}{2}$$

$$LCD = 4$$

$$-2\pi < x - \pi < 2\pi$$

$$-2\pi + \pi < x - \pi + \pi < 2\pi + \pi$$

$$-\pi < x < 3\pi$$

$-\pi$ 3π
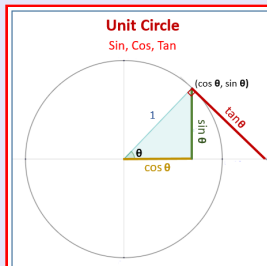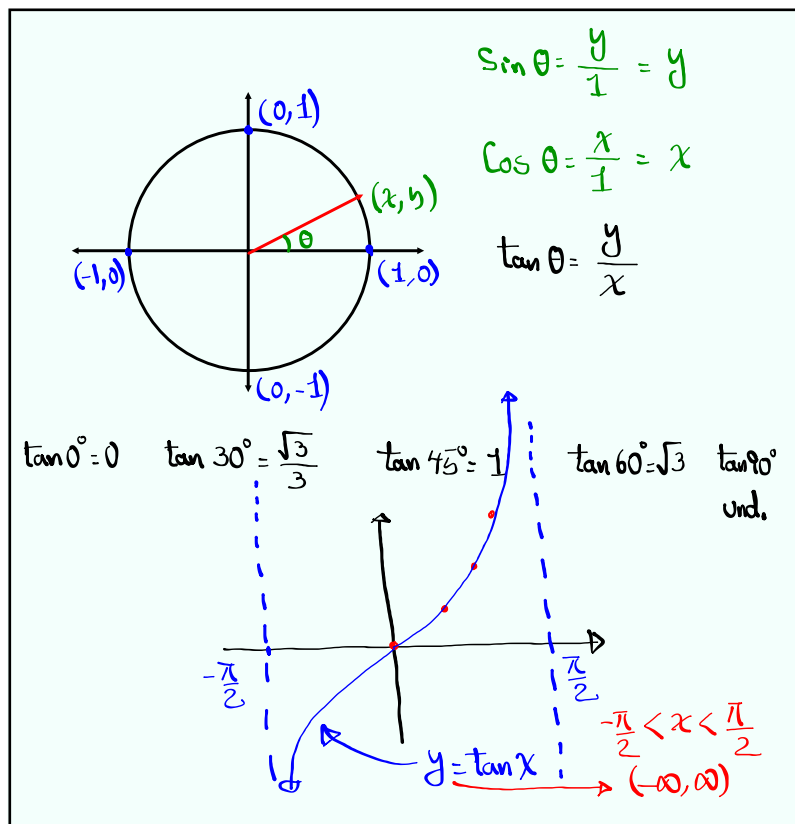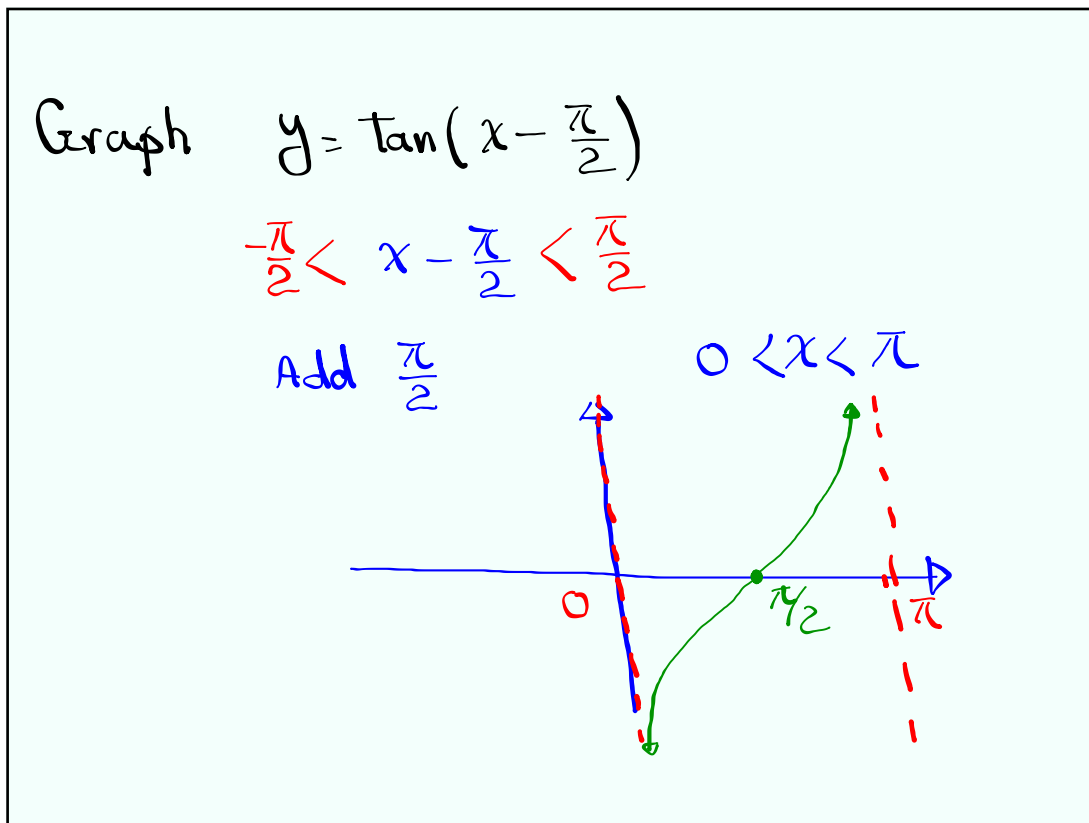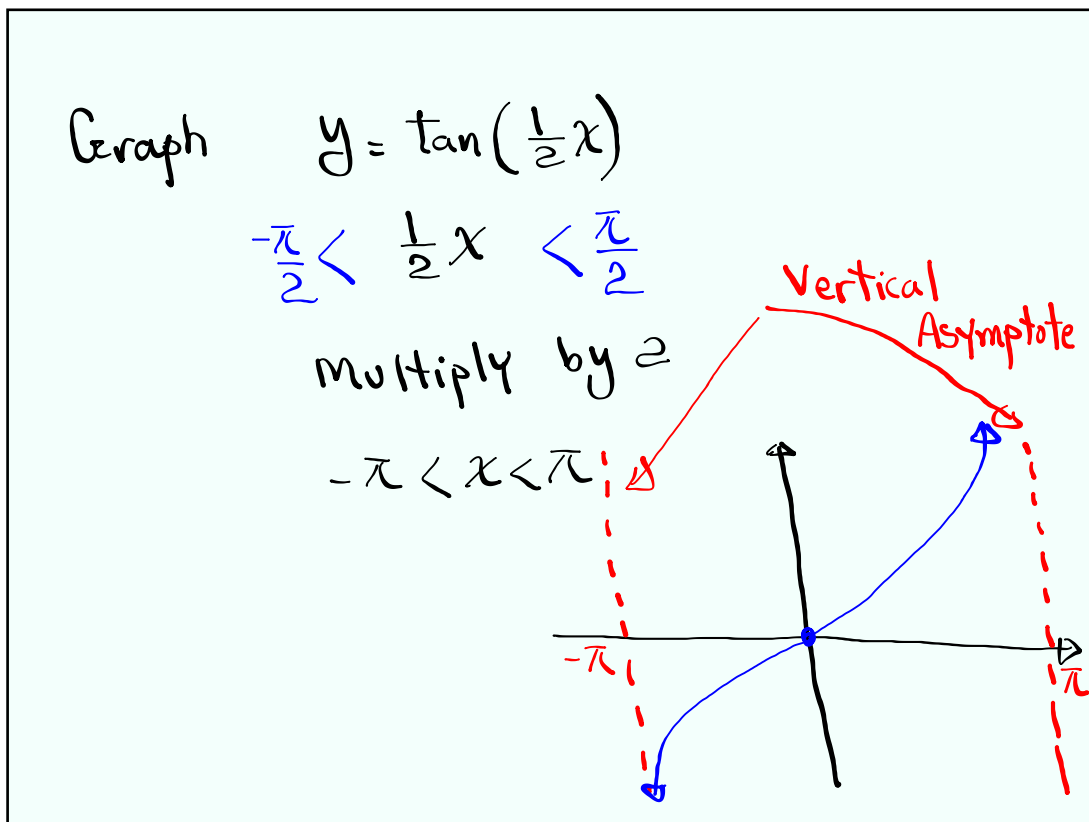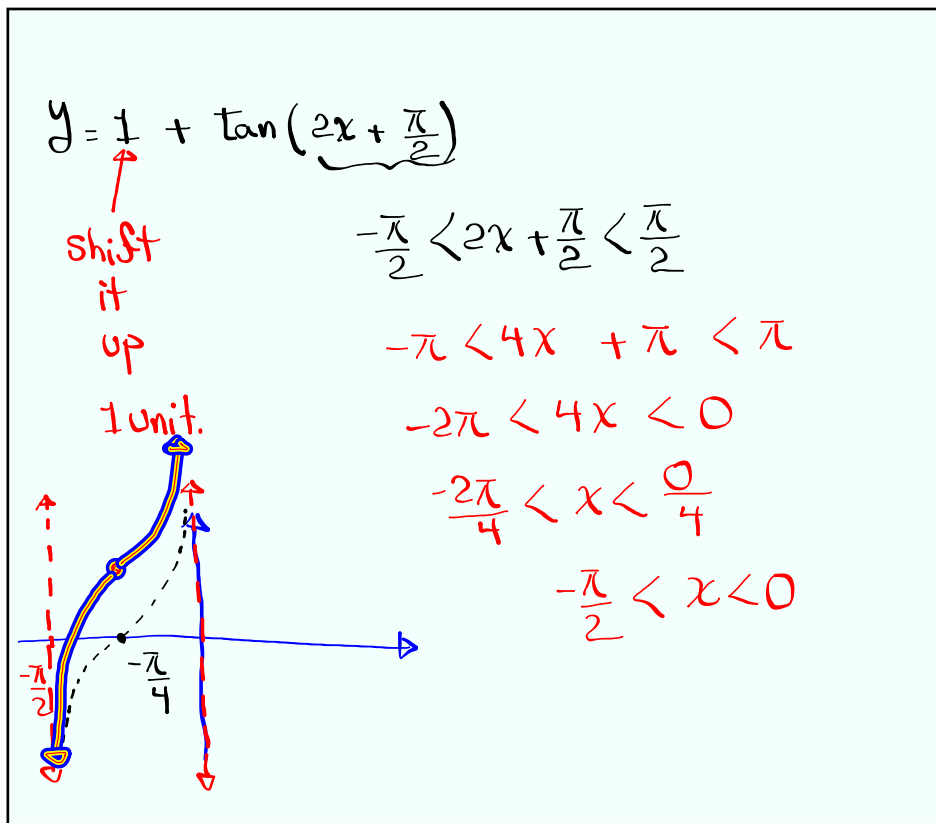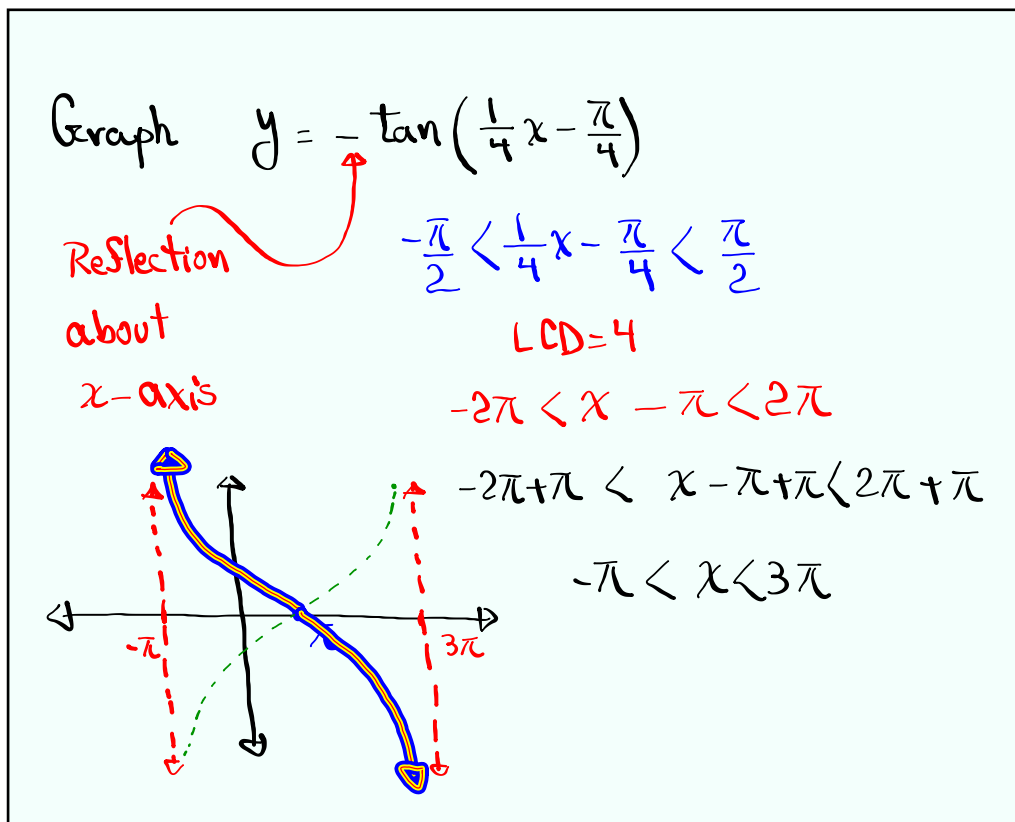
Oct 23-10:49 AM

Graph $y = \tan(-2x - \frac{\pi}{2})$

$y = -\tan(2x + \frac{\pi}{2})$

$-\frac{\pi}{2} < (-2x) - \frac{\pi}{2} < \frac{\pi}{2}$

$-\pi < -4x - \pi < \pi$

$0 < -4x < 2\pi$

$\frac{0}{-4} > \frac{-4}{-4}x > \frac{2\pi}{-4}$

$0 > x > -\frac{\pi}{2}$

$-\frac{\pi}{2} < x < 0$

Let $x = -\frac{\pi}{8}$

$-2x - \frac{\pi}{2} = -2(-\frac{\pi}{8}) - \frac{\pi}{2}$

$= \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$ QIV $\tan < 0$

$\tan(-\frac{\pi}{4}) = -1$

Oct 23-10:54 AM

Graph $y = \tan(-2x - \frac{\pi}{2})$

$y = \tan[-(2x + \frac{\pi}{2})]$

Recall $\tan(-\alpha) = -\tan\alpha$

$y = -\tan(2x + \frac{\pi}{2})$

Reflection about x-axis

$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$

$-\pi < 4x + \pi < \pi$

$-2\pi < 4x < 0$

$-\frac{\pi}{2} < x < 0$

Let $x = -\frac{\pi}{8}$

$2x + \frac{\pi}{2} = 2(-\frac{\pi}{8}) + \frac{\pi}{2}$

$= -\frac{\pi}{4} + \frac{\pi}{2}$

$= \frac{\pi}{4}$

$\tan\frac{\pi}{4} = 1$

$-\tan\frac{\pi}{4} = -1$

Oct 23-11:00 AM

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A boy is rotating a stone at the end of
50 cm string.

He makes (30 revolutions) in 45 Seconds

$30 \cdot 2\pi$

1) Angular speed per Second.
$$W = \frac{\theta}{t} = \frac{60\pi}{45 \text{ Sec.}} = \frac{4\pi}{3} \text{ Rad./sec.}$$

2) Angular speed per minute.
$$W = \frac{4\pi \text{ Rad}}{3 \text{ Sec.}} \cdot \frac{60 \text{ Sec.}}{1 \text{ Min}} = 80\pi \text{ Rad/min}$$

(20 above 60)
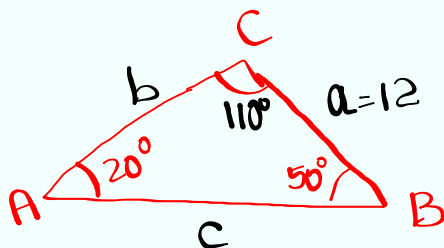
3) Linear Speed.
$$V = \frac{S}{t} \qquad V = r W = 50 \cdot 80\pi$$
$$= 4000\pi \text{ Cm/min.}$$

Oct 23-11:11 AM

In $\triangle ABC$, $A = 20°$, $B = 50°$, $a = 12 \text{cm}$.

Solve triangle ABC.

$$20° + 50° + C = 180°$$
$$C = 110°$$

Law of Sines
$$\frac{Sin 20°}{12} = \frac{Sin 50°}{b} = \frac{Sin 110°}{c}$$

$$b \, Sin 20° = 12 \, Sin 50°$$
$$b = \frac{12 \, Sin 50°}{Sin 20°} = ? \qquad b \approx 27 \text{cm}$$

Oct 23-11:17 AM

How wide is the lake below?

Bearing from Point A to Point B

N 40° W

Bearing from Point A to Point C

N 60° E

distance from A to B is 2.8 miles

"   "   A to C is 3.2 miles

$$x^2 = 2.8^2 + 3.2^2 - 2 \cdot 2.8 \cdot 3.2 \cos 100°$$
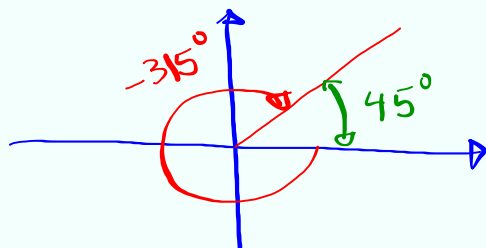
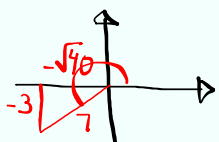$$x^2 = 21.192 \qquad \boxed{x \approx 4.6 \text{ miles}}$$

Draw −315° and mark its ref. angle, then find $\tan(-315°) = +\tan 45°$

$$= \boxed{1}$$
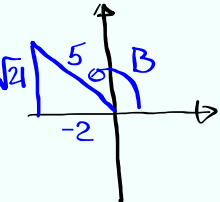
$Sin\ A = \dfrac{-3}{7}$, A is in Q III

$Cos\ B = \dfrac{-2}{5}$ B is in Q II

$90° < B < 180°$

$45° < \dfrac{B}{2} < 90°$ Q I

$Sin\ 2A\ \overset{.}{\varepsilon}\ Cos\ \dfrac{B}{2}$

$Cos\ \dfrac{B}{2} = \pm\sqrt{\dfrac{1+Cos\ B}{2}}$

Q I

$= \pm\sqrt{\dfrac{1+\frac{-2}{5}}{2}} = \sqrt{\dfrac{5-2}{10}} = \sqrt{\dfrac{3}{10}} = \dfrac{\sqrt{3}}{\sqrt{10}} = \dfrac{\sqrt{30}}{10}$

$Sin\ 2A = 2\ Sin\ A\ Cos\ A$

$= 2 \cdot \dfrac{-3}{7} \cdot \dfrac{-\sqrt{40}}{7}$

$= \dfrac{6\sqrt{4}\sqrt{10}}{49} = \dfrac{12\sqrt{10}}{49}$

Oct 23-11:31 AM

$\dfrac{Cos\ 4x + Cos\ 2x}{Sin\ 4x - Sin\ 2x} = \dfrac{2\ Cos\ \frac{4x+2x}{2}\ Cos\ \frac{4x-2x}{2}}{2\ Sin\ \frac{4x-2x}{2}\ Cos\ \frac{4x+2x}{2}}$

$= \dfrac{2\ Cos\ 3x\ Cos\ x}{2\ Sin\ x\ Cos\ 3x}$

$= \dfrac{Cos\ x}{Sin\ x} = \boxed{Cot\ x}$

Oct 23-11:39 AM